TOWARD A QUICK AND REASONABLE ORIENTATION OF THE RECENT ESTABLISHED TRIANGULATION NETS IN EGYPT

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ABSTRACT

Astronomical observations for azimuth are suggested to be carried out using Black's azimuth method, then the geodetic azimuth and the deflection components ξ and η can be calculated with a reasonable standar of accuracy. This has the advantage that astronomical observations for longitude is not required and concequently, it requires only half the observing time of the conventional methods.

1- INTRODUCTION

In the first order base net of Egypt, which goes along the river valley, Laplace stations were established at the terminals of the bases to control the calculated geodetic azimuths. In the years 1979 - 1982 two additional first order triangulation nets were completed in the area between Eastern desert and the Red sea coast. No astronomical observations were taken for these recent established nets. It is well known that astronomical azimuth alone is not enough to control the orientation of a geodetic net unless astronomical observations for longitude and latitude are taken to satisfy Laplace equation in its complete form.

2- FORMATION OF THE BASIC EQUATION FOR GEODETIC AZIMUTH

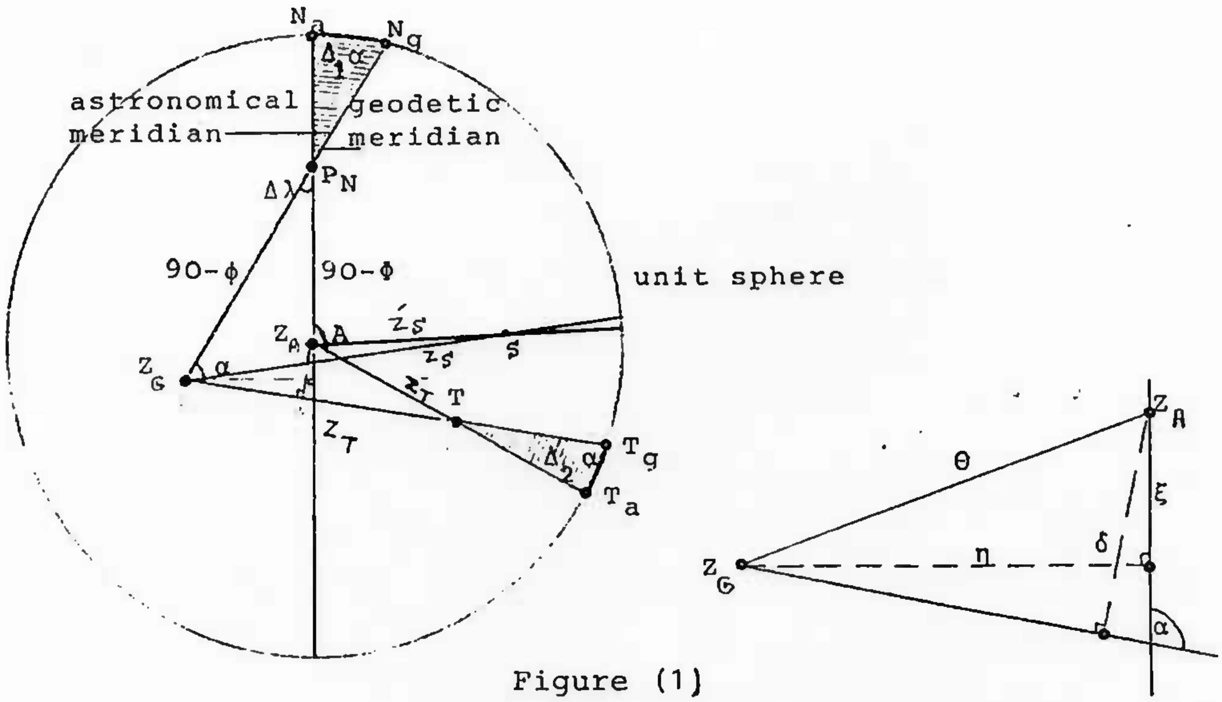

Figure (1)

Figure 1, illustrates a unit sphere with its center at the observation station [3]. Let Z_A denotes the astronomical zenith, and Z denotes the geodetic zenith. The line of sight to the star S, and to the target T for which the azimuth is measured, intersects the sphere at the points S and T respectively. Its zenith distances Z_s', Z_T' and Z_s, Z_T with respect to the zeniths Z_A and Z_G. The point P_N corresponds to the North pole, and the angle at P_N is the difference between the astronomical and geodetic longitude; $\Delta\lambda = \Lambda - \lambda$.

The angles $P_N Z_A S$ and $P_N Z_A T$ correspond to the astronomical azimuth of the star and the target respectively. The angles $P_N Z_G S$ and $P_N Z_G T$ correspond to the geodetic azimuth of the star and the target respectively.

The difference in azimuth $\Delta\alpha$ can be formulated as follows

$$\Delta\alpha = A - \alpha \quad \ldots\ldots (1)$$

where A denotes the astronomical azimuth, and $\Delta\alpha$ consists of two parts [3]

$$\Delta\alpha = \Delta_1\alpha + \Delta_2\alpha \quad \ldots\ldots (2)$$

$$\Delta_1\alpha = \Delta\lambda \sin\varphi = \eta \tan\varphi \quad \ldots\ldots (3)$$

$$\Delta_2\alpha = \delta \cot \bar{Z}_T \doteq \delta \cot Z \quad \ldots\ldots (4)$$

Where

$$\delta = \xi \sin\alpha - \eta \cos\alpha \quad \ldots\ldots (5)$$

The first term $\Delta_1\alpha$ is the same for every target, independent of its azimuth, and zenith distance. The second term $\Delta_2\alpha$ depends on the azimuth and zenith distance, and it is the same as that of an inaccurate leveling of the theodolite.

Consider the horizontal angle from the star to the target at Z_G, Figure 1, is given by

$$SZ_GT = SZ_AT - \delta_T \cot Z_T + \delta_* \cot Z_* \quad \ldots\ldots (6)$$

and the geodetic azimuth of the target is given by

$$A_G^T = A_G^* + SZ_AT - \delta_T \cot Z + \delta_* \cot Z^* \quad \ldots\ldots (7)$$

Now the advantage of the above equation arises from the fact that the geodetic azimuth of the star A_G^* can be calculated by the four parts formula [2] applied to the triangle PZ_GS, when using the method of azimuth by hour angle [1], [4], [6], taking into consideration that the geodetic latitude and longitude of the occupied stations are known. This is the case of the newly established geodetic nets in Egypt, because they are tied to the old nets. Accordingly:

a) The geodetic azimuth of the star is calculated as follows:

$$\cot A_G^* = (-\cos\varphi \tan\delta + \sin\varphi \cos t)\operatorname{cosec} t \quad . . . (8)$$

where

φ = geodetic latitude of the observer,

δ = declination of the star,

t = the hour angle at the time of observation,

= RA - GAST - λ_G

RA = the right assension of the star

λ_G = the geodetic longitude of the observer

GAST= the Greenwich apparent sidereal time

b) The second term $SZ_A T$ is the horizontal angle measured by the theodolite between the star and the target.

c) The third and fourth terms are the usual reduction term $\Delta_2 \alpha$ as in equation (4), and the only unknowns are ξ and η.

From the above discussion it is clear that a minimum of three stars in different azimuths are required to determine A_G, ξ and η. Thus, each star gives an observation equation connecting A_G, ξ and η.

3- DERIVATION OF THE OBSERVATION EQUATIONS

Let us write equation (7) in the following form

$$A_G^T = A_G^* + SZ_A T - (\xi \sin A_T - \eta \cos A_T)\cot Z^T + (\xi \sin A^* - \eta \cos A^*)\cot Z^* \quad . . . . . (9)$$

$$A_G^T = A_G^* + \theta - \xi(\cot Z^T \sin A^T - \cot Z^* \sin A^*) + \eta(\cot Z^T \cos A^T - \cot Z^* \cos A^*) \quad . . . . . (10)$$

Now, if our observations are taken on N stars then let

$\bar{A}$ = mean value of all A_i,

A = most probable value of geodetic azimuth of the target,

a = A - $\bar{A}$ = correction to $\bar{A}$ to give A,

A_i^*= geodetic azimuth of the i^{th} star computed from (8),

A_i= geodetic azimuth of target; i.e. $A_i^* + \theta$,

Z_i^*= zenith distance of the i^{th} star,

Z^T= zenith distance of the target T.

Accordingly,

$$A = a + \bar{A} = A_i - \xi(\cot Z^T \sin\bar{A} - \cot Z_i^* \sin A_i^*) + \eta(\cot Z^T \cos\bar{A} - \cot Z_i^* \cos A_i^*)$$

and the final form of the observation equation will be

$$V = a + \xi(\cot Z^T \sin\bar{A} - \cot Z_i^* \sin A_i^*) - \eta(\cot Z^T \cos\bar{A} - \cot Z_i^* \cos A_i^*) + (\bar{A} - A_i) \quad \ldots\ldots (11)$$

with N observation equations of the above system, the normal equations are formed and solved for the unknowns a , ξ, and η.

4- NORMAL EQUATIONS

The normal equations for the above mentioned system can be given, by taking into consideration that the terms with notation T in equation (11) will be small and can be omitted, as follows:

$$\begin{bmatrix} N & -F & G \\ -F & H & -I \\ G & -I & J \end{bmatrix} \cdot \begin{bmatrix} a \\ \xi \\ \eta \end{bmatrix} = \begin{bmatrix} -K \\ L \\ -M \end{bmatrix} \quad \ldots\ldots\ldots\ldots (12)$$

where

N = number of observations

$F = \sum_{i=1}^{N} \cot Z_i^* \sin A_i^*$

$G = \sum_{i=1}^{N} \cot Z_i^* \cos A_i^*$

$H = \sum_{i=1}^{N} (\cot Z_i^* \sin A_i^*)^2$

$I = \sum_{i=1}^{N} \cot^2 Z_i^* \sin A_i^* \cos A_i^*$

$J = \sum_{i=1}^{N} (\cot Z_i^* \cos A_i^*)^2$

$K = \sum_{i=1}^{N} (\bar{A} - A_i)$

$L = \sum_{i=1}^{N} (\bar{A} - A_i)(\cot Z_i^* \sin A_i^*)$

$M = \sum_{i=1}^{N} (\bar{A} - A_i)(\cot Z_i^* \cos A_i^*)$

5- ACCURACY OF DETERMINING a, ξ, and η :

5-1. Observational errors in the azimuth:

From equation (8) the observational errors arise as a function of the errors in geodetic latitude, declination of the star, and hour angle, can be formulated as follows:

a) Error in A as a function of φ,

By differentiating equation (8) with respect to   ,

$$-\sin t \operatorname{cosec}^2 A \, dA = (\sin\varphi \tan\delta + \cos\varphi \cos t)\, d\varphi$$

$$dA/d\varphi = \frac{-(\sin\varphi \sin\delta + \cos\varphi \cos\delta \cos t)}{\sin t \cos\delta} \sin^2 A$$

$$= \frac{\cos Z \sin^2 A}{\sin t \cos\delta}$$

$$= - \cot Z \sin A \qquad \ldots\ldots\ldots (13)$$

b) Error in A as a function of δ,

By differentiating equation (8) with respect to δ,

$$-\sin t \operatorname{cosec}^2 A \, dA = -\cos\varphi \sec^2\delta \, d\delta$$

$$dA/d\delta = \frac{\cos\varphi \sin^2 A}{\sin t \cos^2\delta}$$

$$dA/d\delta = \sin q / \sin Z$$

$$=(\cos\varphi \sin A)/\cos\delta \sin Z \quad \ldots\ldots\ldots\ldots (14)$$

c) Error in A as a function of t,

By differentiating equation (8) with respect to t,

$$dA/dt = (\sin^2 A/\sin t)(\sin\varphi \sin t - \cot A \cos t)$$

$$= (\sin A/\sin t)(-\cos A \cos t + \sin A \sin t \sin\varphi)$$

$$= \sin A \operatorname{cosec} t \cos q \quad \ldots\ldots\ldots (15)$$

By substituting the well known formula

$$\sin A \cos q = \sin\varphi \sin t - \sin q \cos A \cos Z,$$

in the equation (15), we get

$$dA/dt = \cos\varphi(\tan\varphi - \cos A \cot Z) \quad \ldots\ldots\ldots (16)$$

Then, the total observational error can be written as follows:

$$dA = -\cot Z \sin A \, d\varphi + ((\cos\varphi \sin A)/(\cos\delta \sin Z))d\delta$$

$$+ \cos\varphi(\tan\varphi - \cos A \cot Z)dt \quad \ldots (17)$$

From the above equation (17), it can be seen that;

i) For $A=0^\circ$ or 180° the error arising from $d\varphi$ vanishes,

ii) For $Z=90^\circ$ the error arising from $d\varphi$ vanishes, and that of $d\delta$ and dt will be minimum.

iii) For $q=90^\circ$ or $\delta=90^\circ$, the error arising from dt vanishes

iv) When stars are paired in azimuths; i.e. $A_2 = A_1 \pm 180$ or $A_2 = -A_1$, then the total effect of $d\varphi$, $d\delta$, and dt will vanish by taking into consideration that $Z_2 \sim Z_1$.

5-2. Variances of the unknowns a, ξ, and η

From the normal equation system (12), and if the stars are well balanced [5], then the off-diagonal elements of the normal equation system will vanish and the inverse of it will be simply given as

$$q_{aa}=1/N \quad , \quad q_{\xi\xi}=1/H \quad ,\text{and} \quad q_{\eta\eta}=1/J \quad . . . . .(18)$$

, and the variances of the unknowns will be

$$\delta_a = \pm \delta_o q_{aa}^{1/2} \; , \; \delta_\xi = \pm \delta_o q_\xi^{1/2} \; , \text{ and } \delta_\eta = \pm \delta_o q_\eta^{1/2} \quad . . . .(19)$$

where

δ_o = standard error of an observation of unit weight
= $\left[v^2\right]$ / (n-3).

It can be seen that the proportion of the total information which goes into the determination of each of the unknowns are:

i i) for azimuth $q_{aa}^{-1} / (q_{aa}^{-1} + q_{\xi\xi}^{-1} + q_{\eta\eta}^{-1}) = N/(N+H+J) = \sin^2 Z$

ii) for $\xi = q_{\xi\xi}^{-1} / (q_{aa}^{-1} + q_{\xi\xi}^{-1} + q_{\eta\eta}^{-1}) = \cos^2 Z \sin^2 A$ . . . (20)

iii) for $\eta = q_{\eta\eta}^{-1} / (q_{aa}^{-1} + q_{\xi\xi}^{-1} + q_{\eta\eta}^{-1}) = \cos^2 Z \cos^2 A$

From the above, it can be seen that if it is required to determine a, ξ, and η all with the same accuracy, then let at least a set of four stars of azimuths $\simeq$ 45°,135°, 225°,and 315° are observed. Then by equation (20) the corresponding value of Z will be 35.2° which can be approximated to be within the limits of 30° to 40°.

6- CONCLUSION

From the above analysis it can be seen that the determination of a, ξ, and η may be determined to conventional accuracy $\simeq 0.4''$ [1], by observations of stars of altitude 50° to 60°. This method has the advantage that the astronomical observations for latitude and longitude are not required, and the geodetic values which are known for the new established nets in Egypt are used instead. Accordingly, it requires only the half of the observational time of the conventional methods.

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